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LQR Control of a Flexible Satellite with Movable Mass Actuation

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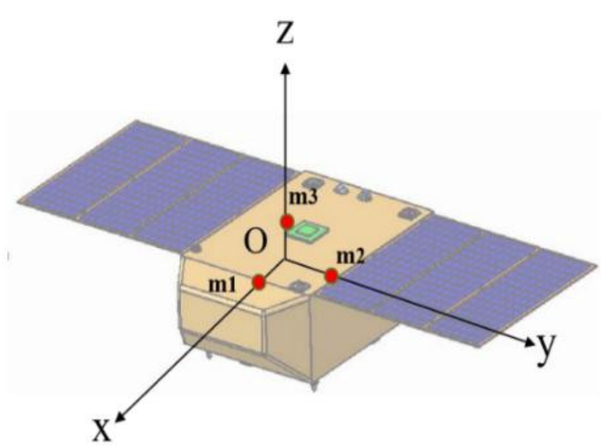
ABSTRACT

This paper addresses the challenging problem of optimal control for a flexible satellite equipped with a movable mass actuator. The dynamics of the satellite system include both rigid body rotations and elastic vibrations due to flexible appendages, which are modelled using second-order differential equations. The primary control input is generated by the strategic movement of an internal mass, which simultaneously produces torques for attitude stabilization and mitigates vibrational energy in the flexible modes. To achieve a delicate balance between minimizing state deviation and control effort, An optimal Linear Quadratic Regulator (LQR) strategy is implemented and its performance is compared with that of a conventional PD controller. The stability of the closed-loop system is theoretically established using the Lyapunov theory. Numerical simulations validate the proposed approach, demonstrating its effectiveness in minimizing flexible mode excitation and maintaining satellite attitude.

Keywords: Attitude Control; Flexible Satellite; LQR Control; Movable Mass.

Graphical abstract



Schematic of ultra-low-orbit spacecraft considering movable mass

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1. Introduction

Satellites with flexible structures, such as large solar panels or antennas, introduce unique control challenges due to the interaction between rigid body dynamics and flexible mode behaviour [1,2]. The motion of these flexible components generates oscillations that can impair pointing accuracy and overall control performance. Traditional control methods that only address rigid body dynamics are often insufficient for managing these oscillations. For successful space missions, it is essential to design control systems that ensure precise pointing and enable rapid maneuvering. Common actuators for these systems include propulsion devices ([3]), angular momentum exchange devices like reaction wheels [4] and control moment gyroscopes (CMGs) [5], and devices interacting with the environment, such as magnetic torquers [6].

In recent years, alternative control strategies involving internal actuators, such as movable masses control (MMC), have been explored. By strategically positioning a movable mass within the satellite, control torques can be generated to manipulate the attitude, Compared to other control actuators, MMC offers three advantages: lightweight, simple structure, and low power consumption [7]. The effects of MMC include center of mass (CoM) shifts, angular momentum exchange, and changes in the inertia tensor, all of which can be effectively leveraged to adjust the spacecraft's state.

Researchers initially explored how shifting internal movable components can modify a spacecraft's energy, thereby altering its state of motion [7,9]. Movable masses have since been applied to various rigid bodies, including underwater gliders [10], warheads [11], and drones [12]. For spacecraft, the primary effect of repositioning movable masses is a shift in the center of mass (CM). This shift can be used to adjust the moment arm of external forces, generating attitude control torques. These external forces can include aerodynamic forces, solar radiation pressure, or thrust from onboard thrusters. Additionally, some researchers have investigated the use of movable masses for three-axis attitude stabilization of low Earth orbit spacecraft [13,14]. In [15], four movable masses positioned on a plane were utilized to achieve three-axis attitude stability. Through specific distribution strategies and coordinated movement of the masses in the same or opposing directions, this setup generated aerodynamic torque or enabled internal momentum exchange torque to maintain stability. To address control methods with coupling effects, equations of motion were derived for an asymmetric rigid spacecraft with a movable mass, and a nonlinear control law was proposed to stabilize a tumbling spacecraft by inducing a simple spin around its major principal axis [16]. Similarly, an optimal control method was developed, employing a single movable mass to reduce the time required for spacecraft detumbling [17].

This paper addresses the challenge of attitude control in flexible spacecraft, where excessive vibration-induced torques can demand control efforts that exceed the reaction wheels' unloading capacity, leading to wheel saturation and compromised attitude control. To address this, we propose an optimal control strategy based on Linear Quadratic Regulator (LQR) principles, designed to stabilize the satellite's attitude and dampen vibrations in the flexible appendages. This approach integrates movable mass control, allowing for adjustments to the spacecraft’s center of mass (CM) relative to its center of pressure (CP) to reduce disturbance torques, thereby improving stability and control efficiency.

This paper is organized as follows: Section 2 describes the mathematical modeling of the satellite's dynamics, including both the rigid body and flexible modes. Section 3 presents the design of the movable mass control system. Section 4 introduces the LQR controller and the optimal control law. Finally, Section 5 presents numerical simulations that compare the proposed control strategy with conventional PD control to demonstrate its effectiveness.

2. Model of flexible satellite attitude Considering Movable Mass

In order to demonstrate the concept of moving mass technology, Figure 1 shows how, when the thrust vector applied to a spacecraft does not intersect the center of mass (CoM), a counterclockwise torque is generated around the CoM, leading to a corresponding change in the spacecraft's attitude. By repositioning internal masses within the spacecraft, the CoM can be adjusted relative to the thrust vector, producing a clockwise torque for attitude control. This adjustment allows the vehicle’s attitude to gradually shift, altering the thrust direction and enabling controlled steering of the spacecraft.

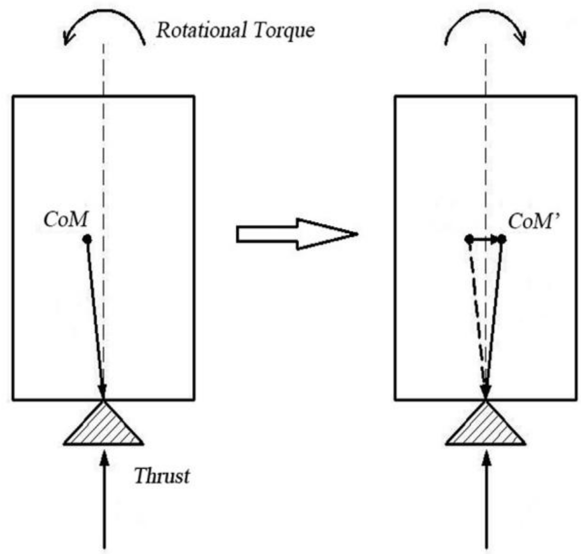

Fig 1. Concept of Moving Mass Technology [19]

2.1. Rigid Body Dynamics

The rigid body dynamics of the satellite are described by Euler's equation, which accounts for the rotational motion around the center of mass:

$$J\dot{\omega}+\omega\times\left(J\omega\right)=T_{ext}+T_{act}+T_{mm} \qquad (1)$$

where:

J , ω , T_{ext} , T_{act} , T_{mm} represents respectively the inertia matrix of the satellite which assumed to be constant, the angular velocity vector, the external torque acting on the satellite, primarily due to the gravity-gradient effect, the satellite's actuators, and the control torque generated by the movable mass system, which we will define in Section 3.

The external torque T_{ext} due to the gravity gradient typically arises when there is an imbalance in mass distribution with respect to the Earth's gravitational field, leading to a restoring torque that aligns the satellite's long axis with the local vertical. In this paper, this effect is considered a disturbance to be mitigated by the control system.

2.2 Flexible Satellite Dynamics

The flexible appendages attached to the satellite introduce additional dynamics due to their vibrational behavior. These dynamics are modeled using the following second-order differential equation:

$$M_f\ddot{q}+C_f\dot{q}+K_f q=F_f(t) \qquad (2)$$

Where:

q and M_f are the modal displacement vector, representing the amplitude of vibration in each mode of the flexible structure and the flexible mode mass matrix, respectively. C_f is the damping matrix, given by

$$C_f=2\zeta\omega_f M_f \qquad (3)$$

where

ζ is the damping ratio, and ω_f is the natural frequency of the flexible mode.

K_f is the stiffness matrix, representing the restoring forces in the flexible appendages. And $F_f(t)$ is the force acting on the flexible structure, which may arise from internal control forces or external disturbances.

The flexible appendages are treated as a distributed parameter system, meaning that their vibrations are described by multiple modes. Each mode has its own natural frequency, damping ratio, and modal mass. The goal of the control system is not only to stabilize the satellite's attitude but also to dampen these flexible mode vibrations.

3. Control from movable mass

The use of a movable mass within the satellite provides an internal method for generating control torques. By moving the mass along a predefined path, the satellite can exert torques to influence its attitude. The control torque generated by the movable mass is expressed as:

$$T_{mm} = \frac{d}{df}\left(r \times mv\right) \qquad (4)$$

where:

r is the position vector of the movable mass relative to the satellite's center of mass. v is the velocity of the movable mass, and m is the mass of the movable component.

The position and velocity of the movable mass can be adjusted dynamically to provide the desired control torque. The concept behind this control approach is similar to that of a reaction wheel. However, instead of spinning a wheel, the system translates a mass within the satellite. The control strategy must account for the dynamics of the movable mass system, as moving the mass also affects the satellite's inertia. This introduces additional complexity into the system model, requiring careful analysis to ensure both stability and performance.

The term $r \times mv$ represents the instantaneous angular momentum generated by internal the mass, by differentiating this term with respect to time. We can find the torque generated by the motion of the mass. Then we obtain

$$T_{mm} = \dot{r} \times mv + r \times m\dot{v} \qquad (5)$$

Where

$\dot{r}$ is the rate of change of the position r (i.e. the velocity of the movable masse relative to the satellite's CoM), and $\dot{v}$ is the acceleration of the movable mass.

4. LQR Control Design

4.1. Objective of LQR Control

The Linear Quadratic Regulator (LQR) is an optimal control technique used to minimize a cost function that balances state errors and control effort. The LQR controller is designed to minimize the following cost function:

$$J = \int_0^\infty \left(x^T Q x + u^T R u\right) dt \qquad (6)$$

where

x is the state vector, including both the rigid body states (attitude and angular velocity) and the flexible mode states (modal displacements and velocities), u is the control input provided by the movable mass system, Q is a weighting matrix that penalizes deviations in the state variables, and R is a weighting matrix that penalizes the control effort.

The goal of the LQR controller is to find an optimal control input that minimizes the cost function. This control input is designed to stabilize the satellite's attitude and suppress flexible mode vibrations while minimizing the control effort.

4.2. *State-Space Representation*

To apply the LQR control, the system dynamics are first represented in state-space form. The state vector is defined as:

$$x = \begin{bmatrix} \theta & \dot{\theta} & q & \dot{q} & r & \dot{r} \end{bmatrix}^T .$$

where:

θ is the angular displacement of the satellite, $\dot{\theta}$ is the angular velocity, q is the modal displacement of the flexible appendages. $\dot{q}$ is the modal velocity, r is the movable mass position , and $\dot{r}$ is the movable mass velocity.

The linearized state-space equations are then written as:

$$\dot{x} = Ax + B_u u \tag{7}$$

where

A is the system matrix and $B_u = [B \quad B_m]$ is the input matrix. These matrices are derived from the rigid body and flexible mode dynamics, including the influence of the movable mass control system which described as follow:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{m}{J} \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -\omega_{flex}^2 & -2\xi_{flex}\omega_{flex} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \ B = \begin{bmatrix} 0 \\ \frac{1}{J} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \text{ and } B_m = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{m} \end{bmatrix}.$$

4.3 Optimal Control Law

The optimal control law is derived as:

$$u = -Kx \tag{8}$$

where

K is the feedback gain matrix, Substituting u into (7) gives the closed loop system :

$$\dot{x} = (A - B_u K)x \tag{9}$$

$A - B_u K$ is the closed loop system matrix ensuring desired stability and performance.
The LQR gain K is computed by solving the following algebraic Riccati equation (ARE):

$$A^T P + PA - PB_u R^{-1} B_u{}^T P + Q = 0 \tag{10}$$

The feedback gain matrix K is then given by:

$$K = R^{-1} B_u{}^T P \tag{11}$$

The LQR controller uses this feedback gain to compute the control input u based on the current state x .

4.4 Lyapunov Stability Proof

Choose a Lyapunov Candidate function based on the positive definite solution P of the ARE:

$$V(x) = x^T P x \tag{12}$$

This function is positive defin
$V(x) > 0$ for all $x \neq 0$, $V(0) = 0$

The time derivative of $V(x)$ is:

$$\dot{V}(x) = \frac{d}{dt}\left(x^T P x\right) = \dot{x}^T P x + x^T P \dot{x} \tag{13}$$

Substitute the Eq(9) gives:

$$\dot{V}(x) = x^T (A - B_u K)^T P x + x^T P (A - B_u K) x \tag{14}$$

According the (ARE) (10) and (11) implies:

$$\dot{V}(x) = -x^T \left(Q + K^T RK\right) x \tag{15}$$

Since $Q > 0$ and $R > 0$, the term $Q + K^T RK$ is positive define, Therefore:

$$\dot{V}(x) < 0 \tag{16}$$

This shows that $V(x)$ is a valid Lyapunov function, and the closed-loop system is globally asymptotically stable.

The flexible modes $(q, \dot{q})$ are included in the state vector x. Their dynamics are penalized in the LQR cost function via the matrix Q, which ensures that the control input suppresses vibrations. By including the modal dynamics in *A* and *B*, the eigenvalues of $A - B_u K$ ensure that the flexible modes are stable and sufficiently damped.

5. Numerical Example

5.1. System Parameters

The following simulation illustrates the dynamic behavior of a satellite system, including rigid-body attitude motion, flexible structural vibrations, and the effects of a movable mass. To stabilize the system we employ a state-space representation and design a Linear Quadratic Regulator (LQR). A comparative PD controller is also implemented to benchmark and demonstrate the effectiveness of the LQR approach. The system's dynamics are simulated over 100 seconds. RMS errors for the attitude angle and angular velocity are computed, showing efficient stabilization with controlled control effort.

We simulate the control of a satellite with the following parameters:

Table 1. Flexible satellite parameters

Inertia (kg.m^2)	440
Flexible mode mass matrix (kg)	20
Damping ratio	0.005
Natural frequency (rad/s)	0.3
Movable mass (kg)	5
Initial position of the movable mass (m)	0.5

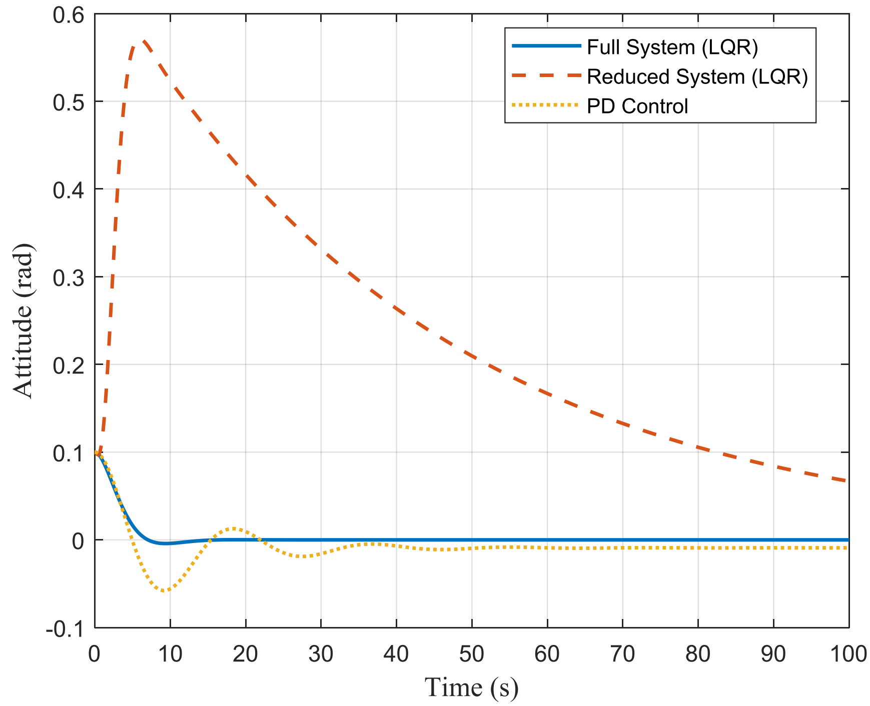

Fig 2. Time response of Pitch Angle

Figure2, compares the attitude response of a satellite system using three control strategies: Full System LQR, Reduced System LQR, and PD Control. The Full System (LQR+ movable mass) demonstrates the best performance, with a fast response, minimal overshoot, and rapid stabilization. In contrast, the Reduced System LQR shows poor performance, characterized by a large overshoot and slow convergence, indicating that simplifying the system compromises control quality. The PD Control provides a moderately effective response with some oscillations and slower settling compared to the full LQR but still achieves acceptable stabilization. Overall, the Full LQR offers superior control, while PD serves as a simpler but less precise alternative.

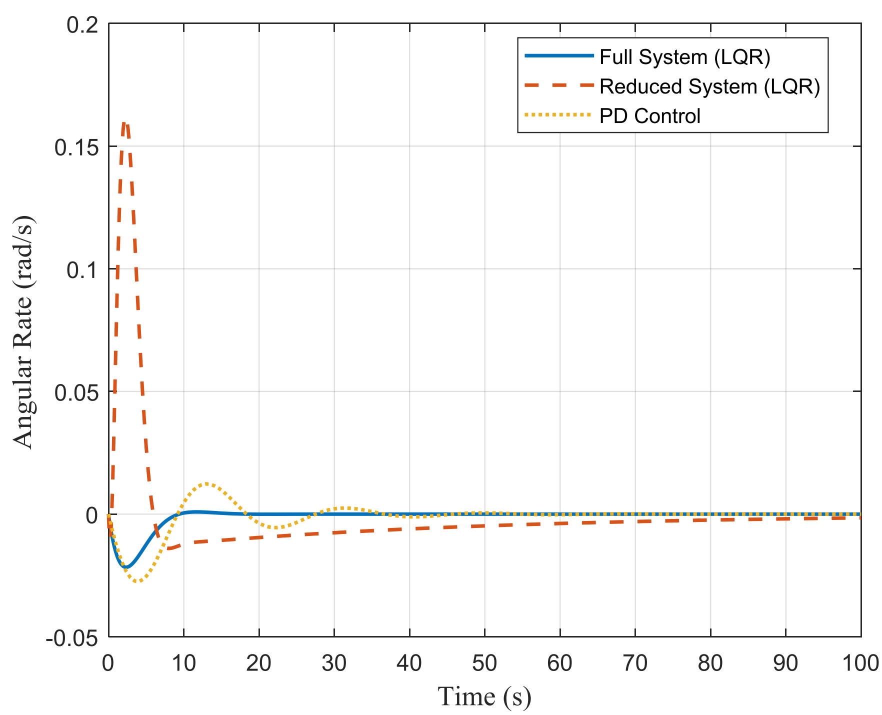

Fig 3. Time response of Pitch rate

The figure 3 illustrates the angular rate response of the satellite system under three control strategies: Full System LQR, Reduced System LQR, and PD Control. The Full System LQR achieves rapid damping with minimal oscillations and quickly stabilizes the angular rate near zero, indicating effective dynamic regulation. The Reduced System LQR exhibits a high initial peak and a sluggish decay, reflecting poor damping and slower stabilization, likely due to the loss of critical dynamics in the model reduction. The PD Control results in oscillatory behavior with moderate overshoot but converges faster than the reduced LQR system. Overall, the Full LQR control again outperforms the others in terms of stability and response speed.

Table 2. RMS errors of the attitude

	Full System (Movable Mass Control)	**Reduced System (Without Movable Mass Control)**	**PD control**
RMS Error of Attitude (rad)	0.0298	0.3318	0.0341
RMS Error of Angular Rate (rad/s)	0.0055	0.0425	0.0065

The reduced system without MMC has an attitude error that exceeds that of the complete system by more than an order of magnitude, demonstrating a marked deterioration in tracking performance and stability. Similarly, the angular velocity error is almost eight times higher, further underlining the central role of the control strategy in maintaining system accuracy. By incorporating moving mass control, the system achieves significant improvements in accuracy and stability, demonstrating the effectiveness of this approach.

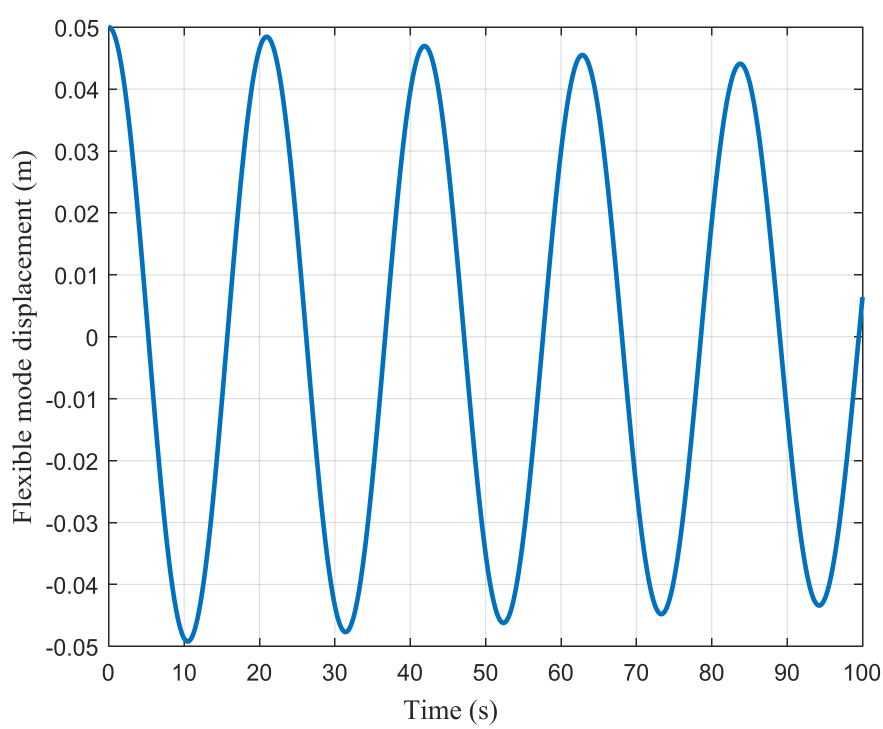

Fig 4. Time response of Flexible Vibration

This figure shows the time evolution of the flexible mode displacement which represents the deformation of the flexible structure. The displacement exhibits sustained oscillations with a slowly varying amplitude, indicating that the flexible mode is damped. This behaviour suggests that the LQR controller focuses mainly on stabilizing the attitude and angular velocity (but only slightly suppresses the dynamics of the flexible mode).

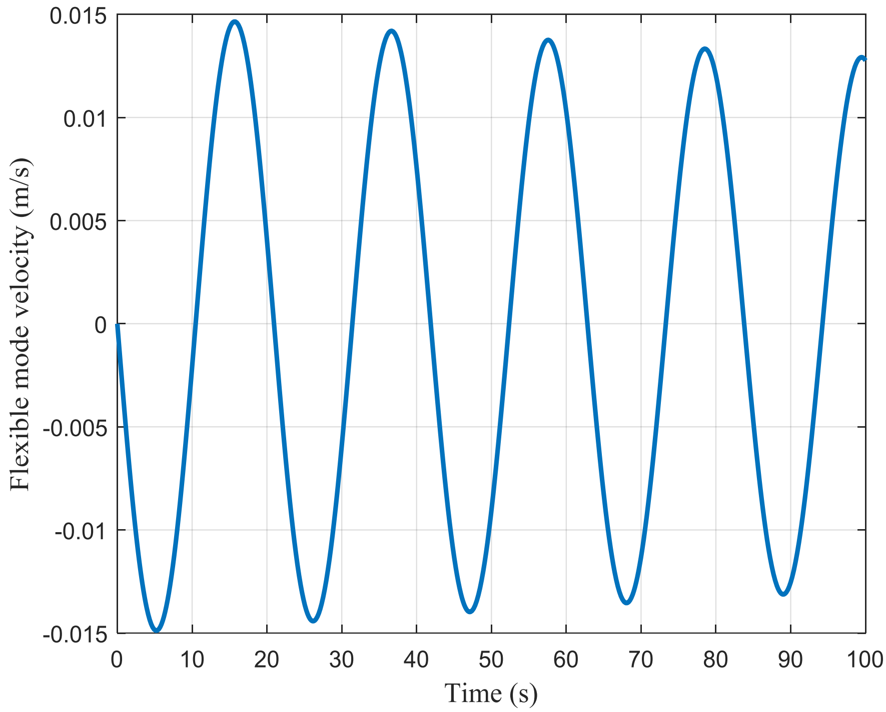

Fig 5, Time response of flexible vibration velocity

Figure 5 illustrates a simulation of the vibration dynamics velocity of the spacecraft's flexible appendages under the proposed control strategy. The periodic oscillations suggest the presence of an underdamped flexible mode, while the relatively stable amplitude indicates that the system is maintaining these oscillations without significant growth or decay, it may demonstrate the control system's ability to regulate and stabilize the flexible mode vibrations, preventing them from escalating.

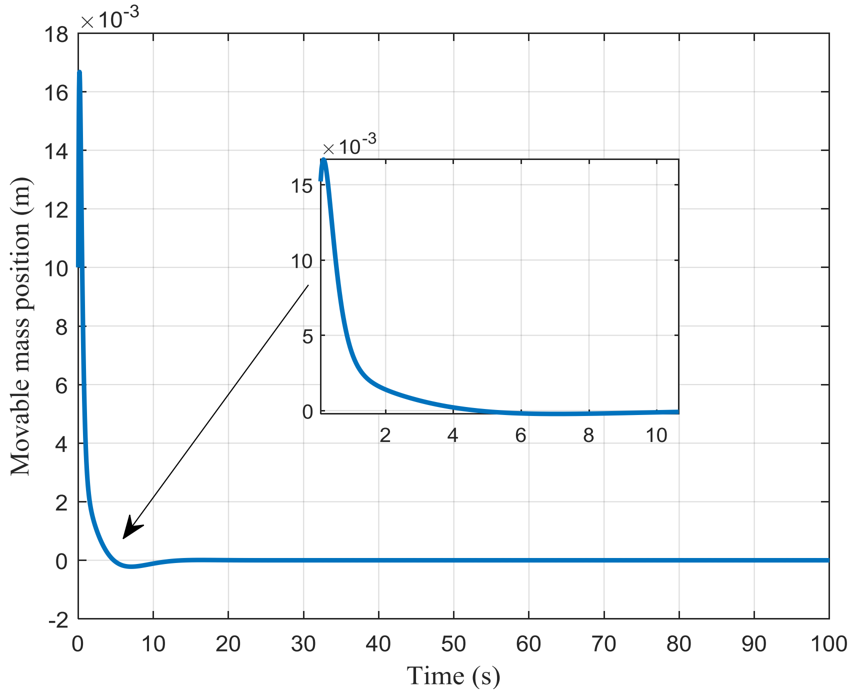

Fig 6, Time response of Movable mass deplacement

This figure represents the time response of the movable mass position in meters as part of the control strategy for the spacecraft. An inset plot zooms in on the initial transient behaviour of the system during the first few seconds.

The initial sharp peak and subsequent decay in the position suggest a transient response of the movable mass as it adjusts to stabilize the system. The rapid settling to a steady-state position indicates that the control system effectively manages the movement of the mass to reduce disturbance torques or vibrations caused by the flexible appendages. The damping in this response highlights the effectiveness of the movable mass mechanism in quickly reaching stability without persistent oscillations.

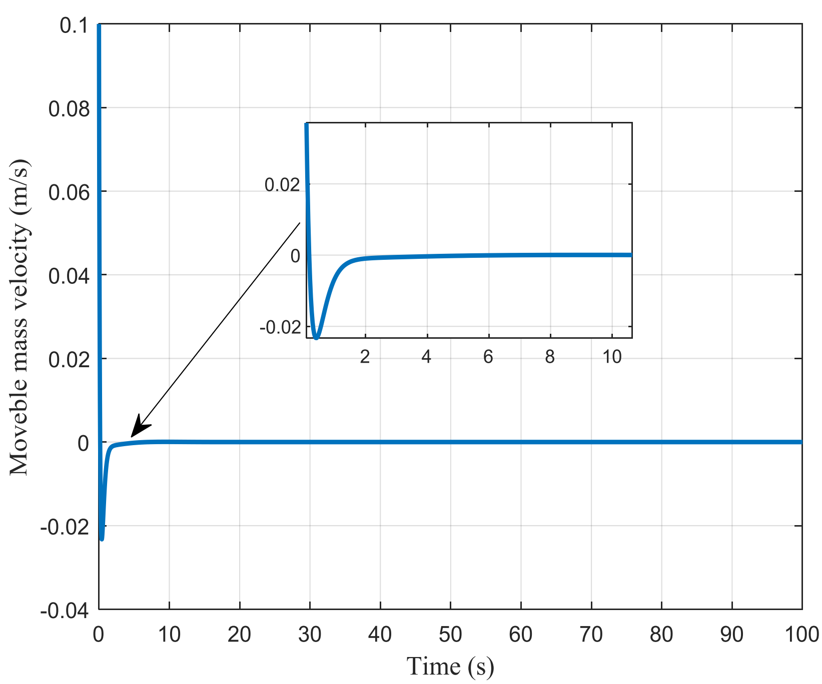

Fig 7. Time response of Movable mass velocity

An initial rapid adjustment in velocity is depicted in figure 7 characterized by a small overshoot, followed by a quick stabilization to near zero velocity. This behaviour indicates that the movable mass undergoes a brief transient motion before settling into a steady state, with minimal oscillation or persistent movement.

The inset focuses on the critical transient phase, where the movable mass velocity peaks and then stabilizes. The quick damping of the velocity demonstrates the effectiveness of the control strategy in ensuring that the movable mass responds rapidly and efficiently to stabilize the system without introducing significant oscillatory.

4. Conclusion

The presented results highlight the effectiveness of the proposed optimal control strategy for attitude stabilization in flexible spacecraft. Through the integration of a Linear Quadratic Regulator (LQR) controller and movable mass control, Comparative analysis against PD control demonstrates superior management of the dynamic interactions between the spacecraft's rigid core and flexible appendages. Indeed, the combined control approach proves effective in stabilizing the spacecraft's attitude while mitigating the adverse effects of vibration-induced torques. These results highlights the potential of this strategy to enhance the performance and reliability of flexible spacecraft, particularly in scenarios where conventional control methods may fail due to wheel saturation or excessive control demands. Future work could explore further optimization of control parameters to improve vibration damping and system efficiency.

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Ethical Statement

This study does not contain any studies with human or animal subjects performed by any of the authors.

Conflict of Interest

The authors declare that they have no conflict of interest.

Data Availability Statement

No applicable.

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